

Magnetized Neutron Star

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1 Introduction

Our main goal in this work is to study magnetized neutron stars by using a fully general-relativity approach presented in the LORENE package¹. Here we have adopted a non-uniform magnetic field profile which depends on the baryon density. This profile has been used in many previous works and seems to be a good choice to explore maximum effects of the internal magnetic field in these objects. Equally important, the magnetic field treated here is poloidal and axisymmetric. The preliminary results show that stars endowed with a strong magnetic field will be deformed and the mass somewhat increased.

2 Formalism

The choice of the coordinates in General Relativity is crucial not only to write the gravitational equations in an advantageous form, but also it can make the problem easier to solve numerically. In the present case, due to the symmetry of the problem, a polar-spherical type coordinates is chosen, namely, the Maximal-Slicing-Quasi-Isotropic coordinates (MSQI) [1]. The metric in the MSQI coordinates is written as:

$$ds^2 = -N^2 dt^2 + A^4 B^2 r^2 \sin^2 \theta (d\phi - N^\phi)^2 - \frac{A^4}{B^2} (dr^2 + r^2 d\theta^2) \quad (1)$$

with $N^\phi(r, \theta)$ the shift vector, $N(r, \theta)$ the lapse function and A and B are functions of r and θ . Details of the gravitational equations can be found in the references [2–4]. The energy momentum tensor of the system reads:

$$T_{\alpha\beta} = (e + p)u_{\alpha\beta} + pg_{\alpha\beta} + \frac{1}{4\pi} \left(F_{\alpha\mu} F_{\beta}^{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} g_{\alpha\beta} \right) \quad (2)$$

where e is the energy density and p the pressure of the fluid. The second term is the electromagnetic contribution and we are not taking into account the magnetic field

¹<http://www.lorene.obspm.fr>

in the equation of state. Work along this line is in progress. Details of the equation of state used in this work can be found in the reference [5].

The magnetic field measured by the Eulerian observer is given by [7]:

$$B_\alpha = -\frac{1}{2}\epsilon_{\alpha\beta\gamma\sigma}F^{\gamma\sigma}n^\beta = \left(0, \frac{1}{A^2Br^2\sin\theta}\frac{\partial A_\phi}{\partial\theta}, -\frac{1}{A^2B\sin\theta}\frac{\partial A_\phi}{\partial r}, 0\right), \quad (3)$$

where $\epsilon_{\alpha\beta\gamma\sigma}$ is the Levi - Civita tensor related to the metric $g_{\mu\nu}$ and n^β the four velocity of the Eulerian observer. Assuming that the matter inside the star has infinite conductivity, the electric field measured by the comoving observer must be zero.

The stress-energy tensor of the magnetic field (second term in eq. 2) is:

$$T_{\alpha\beta}^{EM} = \frac{1}{4\pi} \left(F_{\alpha\mu}F_{\beta}^{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}g_{\alpha\beta} \right) \quad (4)$$

from which one can obtain the sources of the gravitational fields. The energy density reads:

$$E^{EM} = \frac{1+U^2}{8\pi A^8 r^2 \sin^2\theta} (\partial A_\phi)^2, \quad (5)$$

with U the velocity of the fluid in the ϕ direction. The momentum density can be written as:

$$J_\phi^{EM} = \frac{B}{4\pi} \frac{U}{A^6 r \sin\theta} (\partial A_\phi)^2 \quad (6)$$

and the stress 3-tensor components are given by:

$$S_r^{EMr} = \frac{1}{8\pi} \frac{1-U^2}{A^8 r^2 \sin^2\theta} \left[\left(\frac{\partial A_\phi}{\partial r} \right)^2 - \frac{1}{r^2} \left(\frac{\partial A_\phi}{\partial\theta} \right)^2 \right] \quad (7)$$

$$S_\theta^{EM\theta} = \frac{1}{8\pi} \frac{1-U^2}{A^8 r^2 \sin^2\theta} \left[\frac{1}{r^2} \left(\frac{\partial A_\phi}{\partial\theta} \right)^2 - \left(\frac{\partial A_\phi}{\partial r} \right)^2 \right] \quad (8)$$

$$S_\phi^{EM\phi} = \frac{1}{8\pi} \frac{1-U^2}{A^8 r^2 \sin^2\theta} (\partial A_\phi)^2 \quad (9)$$

All the sources term depend on the magnetic vector potential A_ϕ . Besides the above definitions, others important quantities measured by Eulerian observer are the *circunferencial radius* R_{circ}

$$R_{circ} = A^2(r_{eq}, \frac{\pi}{2}) B(r_{eq}, \frac{\pi}{2}) r_{eq}, \quad (10)$$

being r_{eq} the coordinate equatorial radius, the total gravitational mass of the star

$$M = \int \frac{NA^6}{B} (E + S_i^i + \frac{2}{N} N^\phi J_\phi) r^2 \sin \theta dr d\theta d\phi, \quad (11)$$

and an ellipticity which quantifies the apparent oblateness reads

$$\epsilon = \sqrt{1 - \left(\frac{r_p}{r_e}\right)^2}. \quad (12)$$

3 Magnetic field profile

Magnetars are known to possess strong magnetic fields which can be estimated from the the period and the period derivative of the star. These highly magnetized neutron stars are believed to have surface magnetic field of order of $10^{14} - 10^{15}$ G. Besides, the virial theorem states that the magnetic field can be still much higher inside the star than at the surface. Following the references [8 – 16], a non-uniform magnetic field which can depend on the density is parametrized as:

$$B\left(\frac{n_b}{n_0}\right) = B_s + B_0 \left[1 - e^{-\beta\left(\frac{n_b}{n_0}\right)^\gamma}\right]. \quad (13)$$

The parameters are the nuclear density at saturation n_0 , the barion density of matter n_b , the magnetic field on the surface B_s , and a parameter that controls the magnetic field at the center B_0 . We set the parameters β and γ to $\beta = 0.01$ and $\gamma = 2.0$. Other choices for these parameters are possible, but qualitatively the main conclusion remain the same. Usually the value of the magnetic field at the center is about 70% of B_0 . Based on the equations defined in the previous sections, in order to construct our models we need to have an expression for the vector magnetic potential A_ϕ . As we can note in (13) there is no way to get the magnetic potential directly, since the magnetic field is defined as a function of the baryon density n_b . Alternatively, if we suppose a constant magnetic field in the z direction $\vec{B} = B\vec{z}$, the magnetic vector potential is given by:

$$\vec{A}_\phi = \frac{\vec{r} \times \vec{B}}{2} \approx \frac{1}{2} r B \sin \theta \hat{\phi}. \quad (14)$$

Our first approximation is to say that the magnetic field B in equation (14) is locally constant and described by the equation (13) from which we can easily see that the magnetic field is confined inside the star, reaches its maximum value at the center of the star.

To illustrate the approach we show on the Figure 1 isocontours of the magnetic field strength, which lines lie on the surfaces $A_\phi = \text{const.}$ For a specific choice of

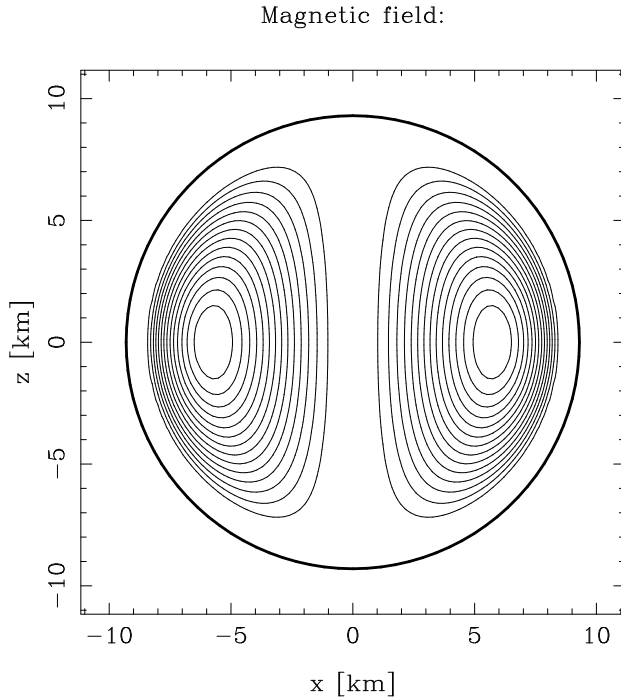


Figure 1: Isocontours of the magnetic field strength in the (x, y) plane

central energy density we have obtained a mass of $M = 1.31 M_{sun}$ and a circumference radius of $R_{circ} = 11.34$ km, whose result are very close to the spherical case and without magnetic field. The ellipticity was found to $\epsilon = 0.032$. This tiny effect is due to *low* value of the magnetic field, namely, $B = 3.5 \times 10^{17}$ G at the center, which is not enough to deform the star. In order to investigate how the mass and the ellipticity, and therefore, the deformation, change with the magnetic field we have evaluated models for different values of B_0 .

Figure 2 shows the change in the mass, while the Figure 3 presents the ellipticity as a function of the magnetic field.

From these figures, both the mass and the ellipticity increase with the magnetic field whose maximum value corresponds to a value of 1.64×10^{18} G at the center of the star. The corresponding ellipticity is found to be $\epsilon = 0.146$, showing that the deformation plays an important role in the global properties of the star. According to the Fig.2 the increasing in the mass is about 7%, less than previous calculation by solving TOV equation, and higher than the 2% predicted by the perturbative approach [16].

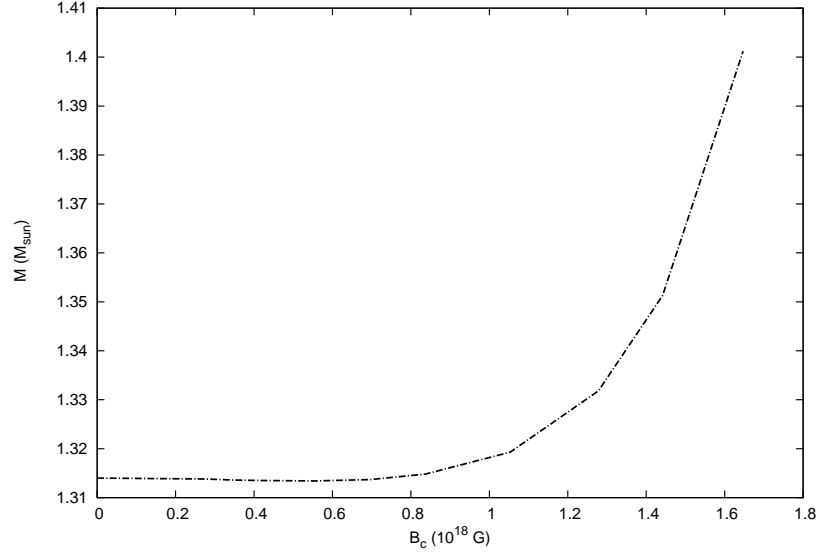


Figure 2: Curve of the gravitational mass M as a function of central magnetic field.

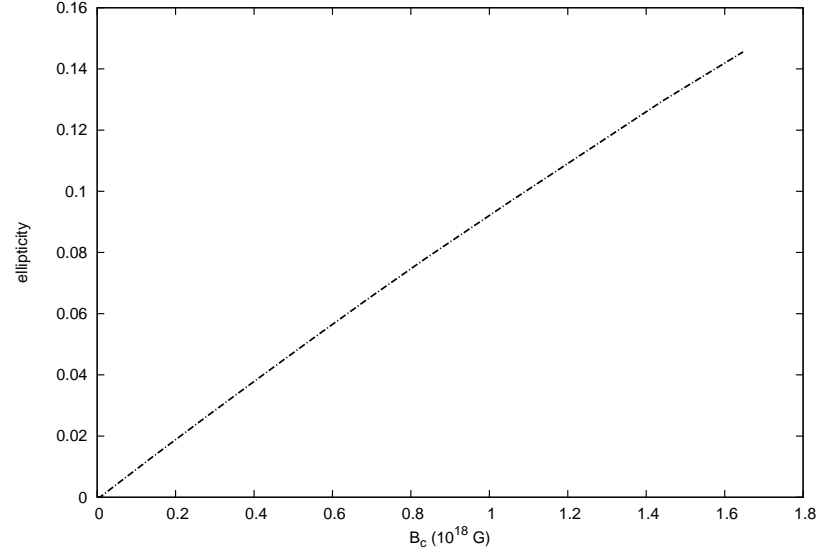


Figure 3: Curve of the ellipticity ϵ as a function of the central magnetic field.

4 Conclusion

We have used the *Lorene package* for computing perfect fluid magnetized stars in general relativity with the inclusion of a non-uniform magnetic field profile. We have then used the code to see the effect of the magnetic field strength on the deformation and also on the mass of the star. The maximum magnetic field found at center is

around $1.64 \times 10^{18} \text{G}$ for a fixed stellar mass of $M = 1.40 M_{\text{sun}}$. The results were obtained for the APR equation of state. A study with a more sophisticated equation of state including magnetic effects and more realistic field configurations is in progress.

Acknowledgement

The authors thanks to Veronica Dexheimer, Joachim Friebe and D. Chatterjee for fruitful comments and discussions. Bruno Franzon wishes to thank the financial support of the CNPq and DAAD.

References

- [1] Gourgoulhon E., 2012, 3+1 Formalism in General Relativity, Lecture Notes in Physics, Springer Verlag
- [2] S. Bonazzola, E. Gourgoulhon, M. Salgado, and J. A. Marck, *Astron. Astrophys.* 278, 421 (1993).
- [3] M. Bocquet, S. Bonazzola, E. Gourgoulhon, and J. Novak, *Astron. Astrophys.* 301, 757 (1993)
- [4] M. Bocquet, S. Bonazzola, E. Gourgoulhon, J. Novak, *Astron. Astrophys.* 301, 757 (1995)
- [5] Akmal, A., Pandharipande, V. R., Ravenhall, D. G., 1998, *Physical Review C*, 58, 1804
- [6] Carter B., 1970, *Comm. Math. Phys.* 17, 233
- [7] Lichnerowicz A., 1967, *Relativistic Hydrodynamics and Magnetohydrodynamics*. Benjamim, New York.
- [8] Bandyopadhyay D, Chakrabarty S and Pal S 1997 *Phys. Rev. Lett.* 79 2176
- [9] D. Bandyopadhyay, S. Chakrabarty, P. Dey, and S. Pal, *Phys. Rev. D* 58, 121301 (1998).
- [10] V. Dexheimer, R. Negreiros, and S. Schramm, *Eur. Phys. J. A* 48, 189 (2012).
- [11] M. Sinha, B. Mukhopadhyay, and A. Sedrakian, *Nucl. Phys. A* 898, 43 (2013).
- [12] G. J. Mao, A. Iwamoto, and Li Zhu-Yia, *Chinese J. Astron. Astrophys.* 3, 359 (2003).

- [13] V. Dexheimer, D. P. Menezes, and M. Strickland, J. Phys. G 41, 015203 (2014).
- [14] Guang-Jun Mao et al 2003 Chin. J. Astron. Astrophys. 3 359
- [15] A Rabhi et al 2009 J. Phys. G: Nucl. Part. Phys. 36 115204
- [16] Mallick, R., Physical Review C 89, 045805 (2014)